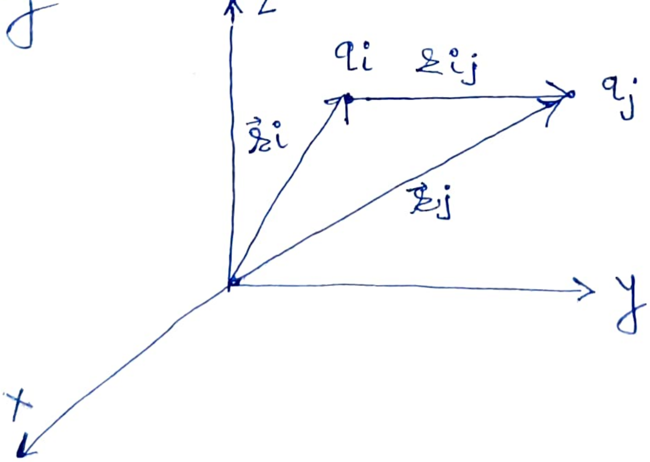
Coulomb's Law

It was found experimently that there is a force of attraction between two oppositely charged bodies and force of repulsion between two simillarly charged bodies.

Coulomb in 1887, gaves a law for the force of attraction or repulsion between two electrically charged bodies separated from each other by a definate distance which is known as Coulomb's law after his name and given below.

The force of attraction or repulsion between two electrical point charges at a fixed distance apart is directly proportional to the product of two point charges and inversly proportional to the square of the distance between them. The direction of the force is always along a straight line joining the two point charges.

Fig



If q_i and q_j are the magnitudes of the point charges separated from each other by a scalar distance r_{ij} then force acting on the charge q_j due to charge q_i is given by

$$\vec{F}_{ij} = \lambda \frac{q_i q_j}{r^2_{ij}} \hat{r}_{ij} \quad \text{------} \quad (1)$$

where $\hat{r}_{ij}$ is the unit vector directed from q_i to q_j and λ is the constant of proportionality whose value is depend upon the medium of separation between the charges. and the units in which various physical quantities are expressed.

In M.K.S. system force F_{ij} is in Newtons, charge is in Coulombs and the constant is written as

$$\lambda = \frac{1}{4\pi\epsilon} \quad \text{------} \quad (2)$$

where ϵ is known as the permitivity of the medium between the charges.

Thus in M.K.S. Coulomb's law may be expressed as

$$\vec{F}_{ij} = \frac{1}{4\pi\epsilon} \frac{q_i q_j}{r^2_{ij}} \hat{r}_{ij} \qquad \hat{r}_{ij} = \frac{\vec{r}_{ij}}{r_{ij}}$$

magnitude $$F_{ij} = \frac{1}{4\pi\epsilon} \frac{q_i q_j}{r^3_{ij}} r_{ij} \quad \text{------} \quad (3)$$

If the medium between the charges is air or vacuum, then $\epsilon = \epsilon_0$ the permitivity of the free space and has the value

$$\epsilon_0 = 8.854 \times 10^{-12} \frac{\text{Coulomb}^2}{N.m^2}$$

N - Newton

So the coulomb's law for forces on the point charges q_j due to charge q_i in free space or air is expressed as

$$F_{ij} = \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}^3} r_{ij} \quad \text{------} \quad (4)$$

It may be noted that coulombs law applies to

(i) The charge q_i and q_j must be static

(ii) This law applies to point charges

The distance r_{ij} between the charge bodies q_i and q_j must be large compared with linear dimensions of the bodies that is q_i and q_j must be point charges.

(iii) This law also applicable to the interaction of elementrary particles such as proton and electrons

(iv) From this law it is also noted that like charges repell each other while unlike charges attract each other.

(v) This law is applicable only when there are only two charges.

If we consider all charges in the universe which are at rest relative to q_j the total electrostatic force on q_j will be the vector sum of forces due to all these charges

$$F_j = \frac{1}{4\pi\epsilon_0} q_j \sum_{i=1}^{n} \frac{q_i}{r_{ij}^3} r_{ij} \quad \text{--- (5)}$$

where the summation is extended over all the charges except j^{th} one i. This is just the principle of superposition for forces which simply state that total force acting on ~~the~~ a point charge is the vector sum of individual forces acting on it.

The above eqn (5) represents the principle of super position of forces.

ebonite or plastic rod — — —
— — —

Fur + + + +
+ + + +

glass rod + + + +
+ + + +

silk — — —
— — —

charge 'q' → scalar

q - may be +ve or -ve

unit Coulomb C large unit of charge

C → 6×10^{18} electron

$1C = 6 \times 10^{18}$ electrons

Electric Field

the region surrounding any charge, In which it has effect of attraction or repulsion on any other charge is known as electric field.

Imagine a small positive test charge q_0 is placed in electric field at the point, it is experience a force $\vec{F}$. Therefore electric field strength or intensity of electric Field $\vec{E}$ at a point in electric is defined as the force experience by unit positive test charge q_0 placed at that point.

Hence $$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{......} (1)$$

$$\vec{F} = q_0 \vec{E}$$

The direction of electric field strength $\vec{E}$ is the direction of Force experience by a positive test charge. Here we have neglected the field produced by the test charge q_0 it self. The defination given by the equation (1) assume that the test charge must be very small so as not to distrub the original electric field.

The electric field intensity is conveni-ently expressed as

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} \quad \text{.....} (2)$$

The limiting process is include in the defination of $\vec{E}$ to insure that the test charge q_0 does not affect the charge distribution which produces $\vec{E}$

The unit of electric field strength or electric field intensity in M.K.S system is Newton per Coulomb or volt/meter.

Ex -
In hydrogen atom the distance between electron and proton is 5.3×10^{-11} m, calculate electrical force of attraction between them

Here

$$q_1 = q_2 = 1.6 \times 10^{-19} C$$

$$r = 5.3 \times 10^{-11} m$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^{9} \frac{N \cdot m^2}{C^2}, \quad \epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{(1.6 \times 10^{-19})^2 \cdot 9 \times 10^{9}}{(5.3 \times 10^{-11})^2}$$

$$= \frac{1.6 \times 1.6 \times 10^{-38} \times 9 \times 10^{9}}{5.3 \times 5.3 \times 10^{-22}}$$

$$= \frac{23.04 \times 10^{-29}}{28.01 \times 10^{-22}} = \frac{23.04}{28.01} \times 10^{-7}$$

$$= 0.8225 \times 10^{-7}$$

$$= 8.225 \times 10^{-1} \times 10^{-7}$$

$$= 8.225 \times 10^{-8} N.$$

Ex – An alpha particle carrying charge of 3.2×10^{-19} C and experiences a force 6.4×10^{-17} N when injected into a uniform electric field. Find the value of $\vec{E}$ E

$$E = \frac{F}{q_0}$$

$$= \frac{6.4 \times 10^{-17}}{3.2 \times 10^{-19}}$$

$$= \frac{64}{32} \times 10^{2} = 2 \times 10^{2}\ N/C$$

$$= 2 \times 10^{2}\ N/C$$

Field due to point charge

According to coulomb's law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{r^2} \cdot \vec{r} \quad \text{-----(1)}$$

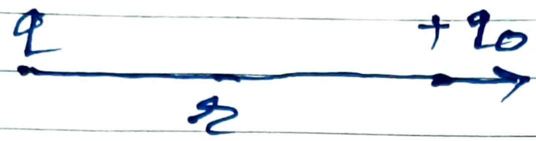
electric field intensity at a distance r due to test charge q_0

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\cancel{q_0} \cdot q}{r^2 \cdot \cancel{q_0}} \vec{r} \quad \text{——(2)}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \vec{r} \quad \text{——(3)}$$

or $\quad E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

Field due to point charge



Consider a positive test charge q_0 is placed at a distance r from the point charge q. According to Coulomb's law, the force experience by a test charge q_0 is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{r^2} \vec{r}$$

where ϵ_0 is the permittivity of free space, $\vec{r}$ is unit vector directed from q to q_0. Therefore the electric field strength $\vec{E}$ at a distance r due to test charge q_0 is

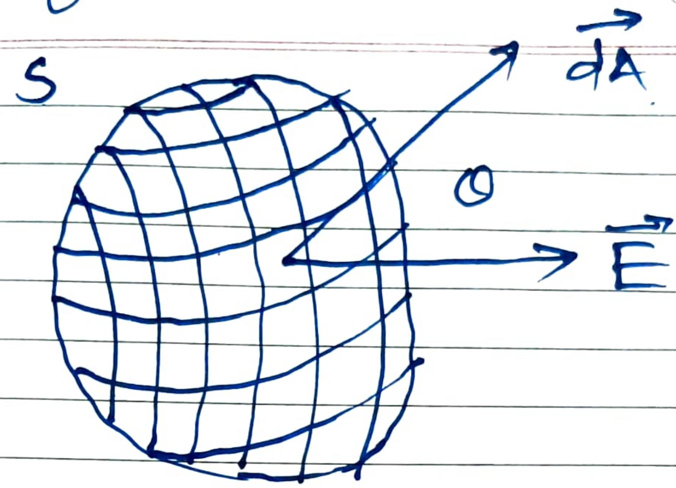
$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \cancel{q_0}}{r^2 \cancel{q_0}} \cdot \vec{r}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \vec{r}$$

If the test charge q_0 is positive the direction of electric field strength is directed always as shown by arrow in the figure.
If test charge is negative the electric field strength is directed towards the charge q

Flux of electric Field

Fig.



Consider the surface S in an electric field $\vec{E}$. Take an element of area dA this element of area is represented by a vector $\vec{dA}$. The direction of $\vec{dA}$ is taken along outward drawn normal to the surface.

The flux of electric field through an element of area $\vec{dA}$ is defined as the product of area $\vec{dA}$ and the component of electric field intensity $\vec{E}$ along the normal to this area. Thus flux of electric $\vec{E}$ through an element area $\vec{dA}$ is given by

$$d\phi = (E\cos\theta)dA = \vec{E}\cdot\vec{dA}$$

Thus flux of electric field intensity $\vec{E}$ through an element of area $\vec{dA}$ is a scalar product of $\vec{E}$ & $\vec{dA}$.

The total flux through the whole surface area S is obtained by adding up the scalar quantity $\vec{E}\cdot\vec{dA}$ for all elements of area into which surface has been devided classmate

Date ______
Page ______

thus total flux

$$\phi = \sum_{s} \vec{E} \cdot d\vec{A}$$

replacing the sum over the surface by integration we get

$$\phi = \int_{s} \vec{E} \cdot d\vec{A}$$

In the close surface, the flux of electric field is

$$\phi = \oint_{s} \vec{E} \cdot d\vec{A}$$

The surface integral denotes that the surface has to be devided into very small element of the area $d\vec{A}$

DATE

Gauss's Law

Statement

Total flux of electric field through the closed surface is equal to $\Sigma q/\epsilon_0$

OR

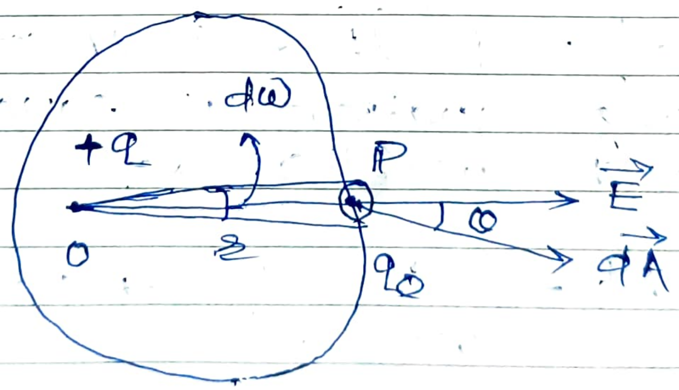
Total flux of electric field through the closed surface is equal to total charge enclosed by the surface devided by ϵ_0

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

where q is the ~~total~~ charge enclosed by the surface through which the flux is calculated.

Proof:-

Fig- Flux due to internal charges



Suppose a charge q is placed at point O inside the closed surface. Take a point P on the surface and consider small area $d\vec{A}$ on the surface around P

Let OP = r

The electric Field at point P on the surface

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q\, q_0}{r^2} \cdot \frac{\vec{r}}{q_0}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \vec{r}$$

or its magnitude

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad \text{———— (1)}$$

along line OP; suppose line OP makes an angle θ with the outward normal to $\vec{dA}$

The flux of electric field through $\vec{dA}$ is

$$d\phi = \vec{E} \cdot \vec{dA} = E\, dA \cos\theta \quad \text{———— (2)}$$

using equation (1) equation (2) can be written as

$$d\phi = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r^2}\, dA \cos\theta \quad \text{———— (3)}$$

we know that the solid angle subtended by surface dA at point O

$$d\omega = \frac{\text{surface area}}{(\text{radius})^2}$$

12

DATE

$$\therefore d\omega = \frac{dA\cos\theta}{r^2}$$

$\therefore$ equation (3) may be written as

$$d\phi = \frac{q}{4\pi\epsilon_0} \cdot d\omega \quad \text{------(4)}$$

The total flux through the whole of closed surface we integrate above equation (4)

$$\oint d\phi = \frac{q}{4\pi\epsilon_0} \cdot \oint d\omega$$

The total solid angle subtented by the surface at internal point is 4π hence

$$\phi = \frac{q}{\cancel{4\pi}\epsilon_0} \cdot \cancel{4\pi}$$

$$\phi = \frac{q}{\epsilon_0} \quad \text{------(5)}$$

Here q is net charge inside the close surface

$$\therefore \phi = \oint \vec{E} \cdot \vec{dA} = \frac{\Sigma q}{\epsilon_0}$$

If there are number of charges q_1, q_2 ..... present inside the surface, then

$$\phi = \oint \vec{E} \cdot \vec{dA} = \frac{(q_1 + q_2 + \dots)}{\epsilon_0} \quad \text{------(6)}$$

DATE

If the charge lies out side the sphere

i'e If there is no charge inside the close surface then flux through the close surface is zero

$$\phi = \oint \vec{E} \cdot d\vec{A} = 0 \quad \text{------} \quad (7)$$

Ex - A sphere of radius 5 cm has a point charge $q = 17.7\ \mu C$ located at the center. Find the electric flux through it.

According to Gauss law

$$\phi_E = \frac{q}{\epsilon_0}$$

$$q = 17.7\ \mu C = 17.7 \times 10^{-6} C$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$$

$$\phi_E = \frac{17.7 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= 2 \times 10^{6} \frac{C}{C^2/Nm^2} \quad \cancel{C^2} \cancel{N^{-1}}$$

$$= 2 \times 10^{6} N m^2 C^{-1}$$

13

classmate

Date ______

Page ______

Differential form of Gauss law

To prove $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

Integral form of Gauss law

$$\oint \vec{E} \cdot d\vec{A} = q/\epsilon_0 \quad \text{———} (1)$$

to convert it into diff. Form we make the use of Gauss divergence thm.

$$\oint_S \vec{A} \cdot d\vec{s} = \iiint_V (\vec{\nabla} \cdot \vec{A})\, dv \quad \text{———} (2)$$

For an electrostatic Field $\vec{E}$

$$\oint_S \vec{E} \cdot d\vec{s} = \iiint_V (\vec{\nabla} \cdot \vec{E})\, dv \quad \text{———} (3)$$

Gauss Thm. can be written as

$$\oint_S \vec{E} \cdot d\vec{s} = \iiint_V (\vec{\nabla} \cdot \vec{E})\, dv = \frac{q}{\epsilon_0} \quad \text{———} (4)$$

if ρ is vol. charge density at a point, then $q = \iiint_V \rho\, dV$

Devided by ϵ_0 throughout we get

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \iiint_V \rho \cdot dv \quad \text{———} (5)$$

classmate

Date

Page

comparing eqⁿ (4) & (5) we get

$$(\vec{\nabla} \cdot \vec{E}) = \frac{\rho}{\epsilon_0} \quad \cdots (6)$$

is the differential form of Gauss law.

15

classmate
Date / /
Page

Electric potential

Electric Field of a charge body can be describe in two ways, either as electric field strength $\vec{E}$ or interms of electric potential V. At any point in space these two quantities are equ related to each other.

Consider the two points A and B in space. If the test charge q_0 is move from A to B, the amount of work is done in moving the charge from A to B is given by

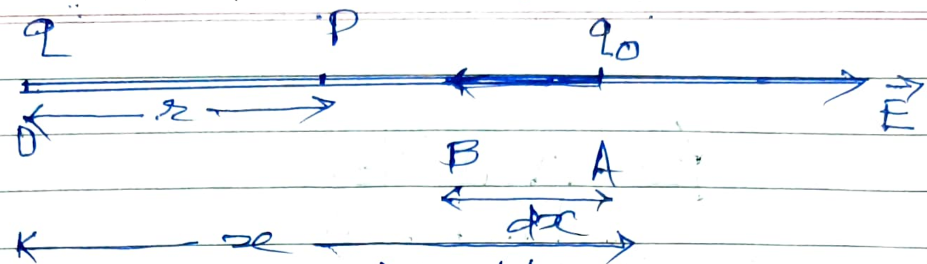
$$W_{AB} = q_0 (V_B - V_A) \quad \text{———— (1)}$$

V_A and V_B are the potentials at point A and B respectively

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

$$dV = \frac{W_{AB}}{q_0} \quad \text{- - - - - (2)}$$

W_{AB} is the work done by the electrostatic force on the test charge q_0 when it move from A to B. The unit of electric potential is joule per coulomb or is called volt. dV in the equation (2) is called potential difference between two points.



By using coulomb's law magnitude of force exerted on the test charge q_0 due to the charge q is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{x^2} \text{ ——— } (3)$$

Magnitude of electric field intensity at point A

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q \cancel{q_0}}{x^2 \cancel{q_0}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \text{ ——— } (4)$$

work done to displace a unit positive test charge q_0 from A to B through a small distance dx against the field is given by

$$dW = - E \cdot dx$$

From eqn. (4) above equation we can write

$$dW = - \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2} \cdot dx$$

Total work done to displace a unit positive test charge q_0 from

17

Date 3

~~Flux~~

~~Electric field through the given surface.~~

infinity to point P is obtain by integrating above equation between limit ∞ to r

$$W = \int_{\infty}^{r} dw = \frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} -\frac{1}{x^2}\, dx$$

but the work done is equal to electric potential at point P

$$V = \frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} -x^{-2}\, dx$$

$$= \frac{q}{4\pi\epsilon_0} \left[-\frac{x^{-2+1}}{-2+1} \right]_{\infty}^{r}$$

$$= \frac{q}{4\pi\epsilon_0} \left[x^{-1} \right]_{\infty}^{r}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r} \quad \text{---} \quad (5)$$

is the electric pot. at point P at a distance r from q. The electric pot. is +ve or -ve depending upon sign of charge.

Ex – Eight charges of the value $3\mu C$, $5\mu C$, $7\mu C$, $10\mu C$, $15\mu C$, $-1\mu C$, $-5\mu C$ and $-7\mu C$ are placed symmetrically on the circle of radius 0.4m in air. Calculate potential at the center of circle.

Sol^n. $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

$$= \frac{9\times10^{9} \times 27\times10^{-6}}{0.4}$$

$$\Sigma q = (3+5+7+10+15-1-5-7)\mu C$$

$$= 27\mu C = 27\times10^{-6} C.$$

$$V = \frac{9\times27}{0.4}\times10^{3}$$

$$= \frac{243}{0.4}\times10^{3}$$

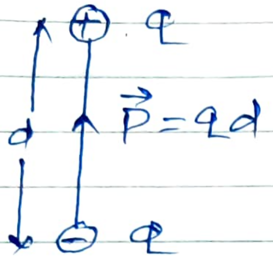
$$= 607.5\times10^{3}$$

$$= 6.075\times10^{5}\ V.$$

19

Electric Dipole & Field due to

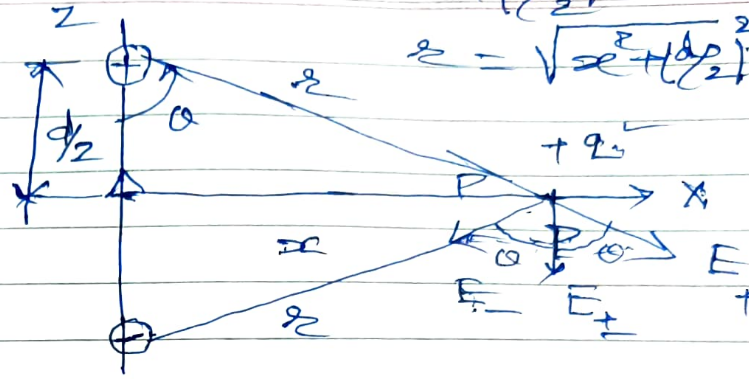
Ele. dip.



Ele. dip.

$r^2 = x^2 + (d/2)^2$

$r = \sqrt{x^2 + (d/2)^2}$



Ele. Field at Point P due to dipole

Eg Na^+Cl^- H_2O, HCl

$\vec{P} = qd$ ——— (1)

$\vec{E} = \vec{E}_+ + \vec{E}_-$

$E_+ = E_- = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$ ——— (1)a

$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{[x^2 + (d/2)^2]}$ ——— (2)

x component

$E_+ \sin\theta - E_- \sin\theta = 0$

z. component of electric field.

$E = E_+ \cos\theta + E_- \cos\theta$

$= 2E\cos\theta$ ——— (3)

From the Fig $\cos\theta = \frac{d/2}{\sqrt{x^2 + (d/2)^2}}$

substitute this value and eqⁿ (2) in eqⁿ (3)

$E = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{[x^2 + (d/2)^2]} \cdot \frac{d/2}{[\sqrt{x^2 + (d/2)^2}]}$

20

$P = qd \quad , \quad q = \frac{P}{d}$

$$E = \cancel{q} \cdot \frac{1}{4\pi\epsilon_0 \left[x^2 + (d/2)^2\right]} \cdot \frac{\frac{P}{\cancel{d}} \cdot \frac{\cancel{d}/2}{}}{\left[x^2 + (d/2)^2\right]^{1/2}}$$

$$= \frac{P}{4\pi\epsilon_0} \cdot \frac{1}{\left[x^2 + (d/2)^2\right]^{3/2}} \quad \text{---} (4)$$

$$= \frac{P}{4\pi\epsilon_0} \cdot \frac{1}{x^3\left[1 + (d/2x)^2\right]^{3/2}}$$

$$= \frac{P}{4\pi\epsilon_0} \cdot \frac{1}{x^3} \left[1 + (d/2x)^2\right]^{-3/2} \quad \text{---} (5)$$

to solve equation by using Binomeal expansion

$$(1+y)^n = 1 + ny + \frac{n(n-1)}{2!} y^2 + \cdots$$

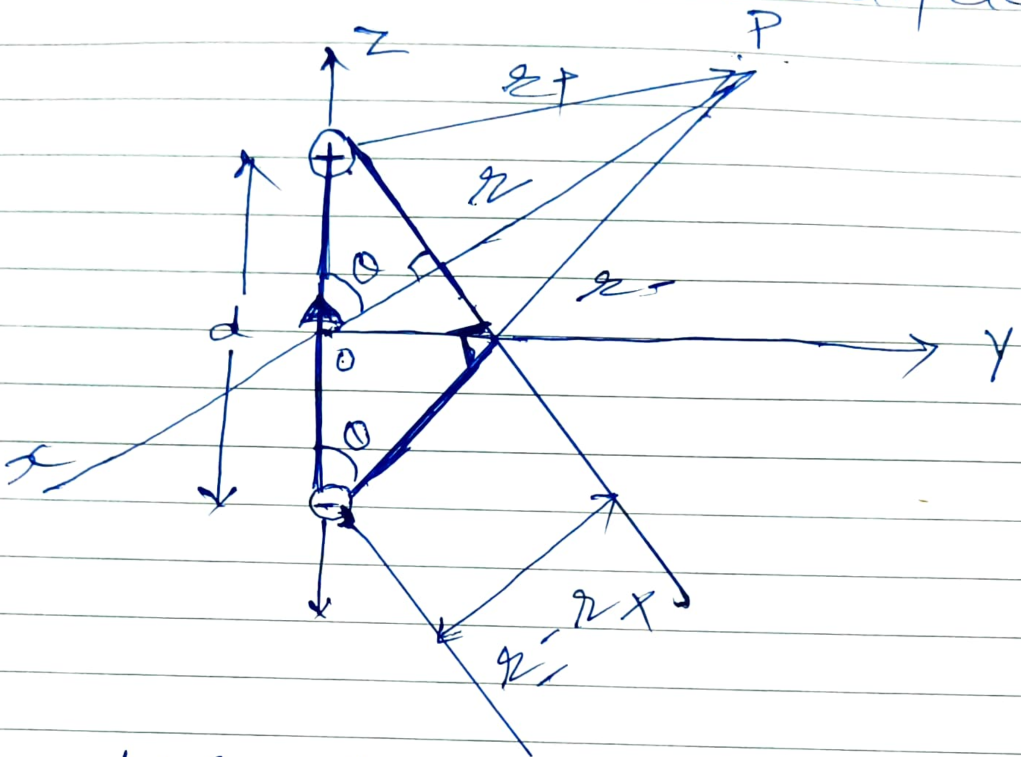
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{P}{x^3} \left[1 - \frac{3}{2}\left(\frac{d}{2x}\right)^2 + \cdots\right]$$

$x >> d$ it is sufficient to keep 1^{st} term only

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{P}{x^3}} \quad \cdots (6)$$

4

Potential due to electric dipole



$$d + r_+ = r_-$$

$$d = r_- - r_+$$

$$\cos\theta = \frac{r_- - r_+}{d}, \quad r_- - r_+ = d\cos\theta$$

In the Fig. we place the origin of co-ordinate system at the center of dipole. We seek the electric potential at point P at a distance r from the center of dipole at an angle θ from the axis of dipole (z-axis)

The distance from positive and negative charges to point P are r_+ and r_- respectively

We know that $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ —— (1)

$$V = \frac{1}{4\pi\epsilon_0} \cdot \left[\frac{q}{r_+} + \frac{-q}{r_-} \right] \text{ —— (2)}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \left[\frac{q r_- - q r_+}{r_+ r_-} \right] \text{ —— (3)}$$

$r >> d$ then in this case

$$\cos\theta = \frac{r_- - r_+}{d}$$

$$r_- - r_+ = d\cos\theta$$

$$r_+ \cdot r_- = r^2$$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot d\cos\theta}{r^2}$$

but $P = qd$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \cdot \frac{P\cos\theta}{r^2}} \text{ - - - - (4)}$$

This equation gives potential at point P due to dipole.

23

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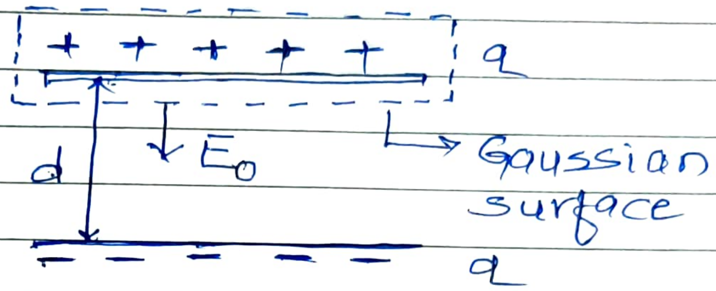
Gauss's Law in dielectrics

Fig. 1



The electric Field E_0 at any point on the Gaussian surface is given by

$$\oint E_0 \cdot ds = \frac{q}{\epsilon_0}$$

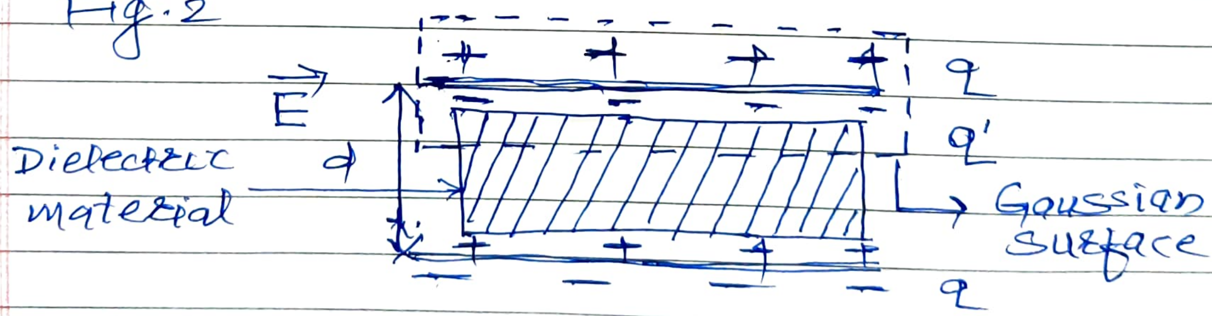
$$E_0 A = \frac{q}{\epsilon_0}$$

$$E_0 = \frac{q}{\epsilon_0 A} \quad \text{——— (1)}$$

$\because$ $E_0 \parallel ds \Rightarrow \because \oint E \cdot ds = E\, ds \cos\theta$

$\theta = 0^\circ$ $\oint E_0 \cdot ds = \oint E_0\, ds = E_0 A$

Fig. 2



Net charge within the Gaussian surface is $(q - q')$ where q' is the induced charges on the surface of dielectric.

The electric Field E is given by

$$\oint E \cdot ds = \frac{(q - q')}{\epsilon_0} \quad \text{——— (2)}$$

$$EA = \frac{q - q'}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \text{ ———— } (3)$$

Since $E < E_0$

If V_0 and V_d is the potential diff. without and with dielectric, then

$$\frac{E_0}{E} = \frac{V_0}{V} = K,$$ K is dielectric const.

$$\frac{1}{E} = \frac{K}{E_0}$$

$$E = \frac{E_0}{K} \text{ - - - - - - - } (4)$$

From equation (1) above equation can be written as

$$E = \frac{q}{\epsilon_0 A} \cdot \frac{1}{K} \text{ - - - - } (5)$$

equating equation (3) & (5) we get

$$\frac{q}{\cancel{\epsilon_0 A}} \cdot \frac{1}{K} = \frac{q}{\cancel{\epsilon_0 A}} - \frac{q'}{\cancel{\epsilon_0 A}}$$

$$\frac{q}{K} = q - q'$$

multiply by -ve throughout

$$-\frac{q}{K} = -q + q'$$

PTO

$$q' = q - \frac{q}{K}$$

$$q' = q\left(1 - \frac{1}{K}\right) \quad \text{-----} (6)$$

This shows that induced charge q' is ~~than~~ less than the Free charge q and is zero when $K = 1$

$$q' = q\left(1 - \frac{1}{K}\right) = q(1-1) = 0$$

Keeping the value of q' from eq^n. (6) in eq^n. (2)

$$\epsilon_0 \oint E \cdot ds = q - q'$$

$$\epsilon_0 \oint E \cdot ds = q - q\left(1 - \frac{1}{K}\right)$$

$$\epsilon_0 \oint E \cdot ds = \cancel{q} - \cancel{q} + \frac{q}{K}$$

or

$$\boxed{\epsilon_0 \oint K E \cdot ds = q} \quad \text{———} (7)$$

This is Gauss law in the presence of dielectric. Here we see that ~~there~~ integral contains factor K and effect of induced surface charge q' is ignored by taking account K the dielectric constant.

Dielectrics

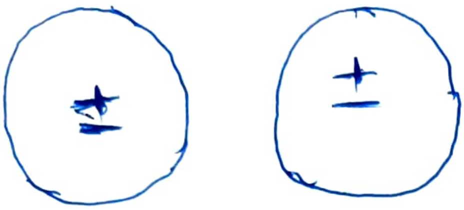
Substances are classified on the basis of their electrical behavior into metals, semiconductors and insulators. The substance through which electrical charges or charge carriers are free to move are called metals. In this, charge carriers are electrons. The substances do not passage of electric current are called insulators or dielectrics. However in insulator local microscopic displacement may takes place. Actually there are no perfect insulators.

Eg. glass, ceramic, mica, asbetos, wood, resin and rubber.

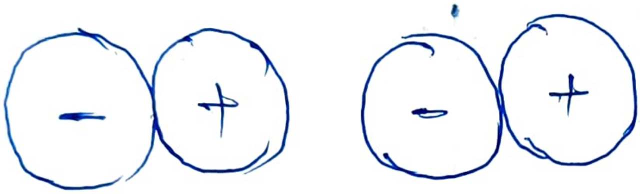
In insulators, there are no free electrons or number of free electrons are too low. These electrons are tightly bound to the nucleus of an atom. When the potential difference is applied to an insulators no electric current flow, but there is slight re-arrangement of electric charges within the atoms. The presence of electric field may change the behavior of an insulator are called dielectrics.

Non Polar and Polar Molecules

Non polar Molecules



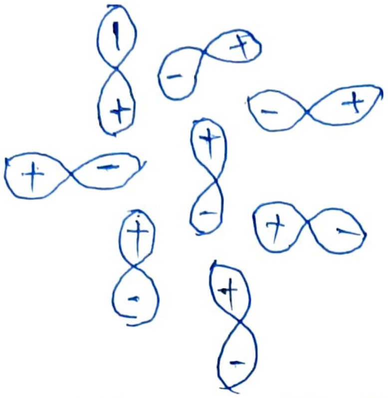
dip. mom = 0
~~Non polar~~ $E = 0$



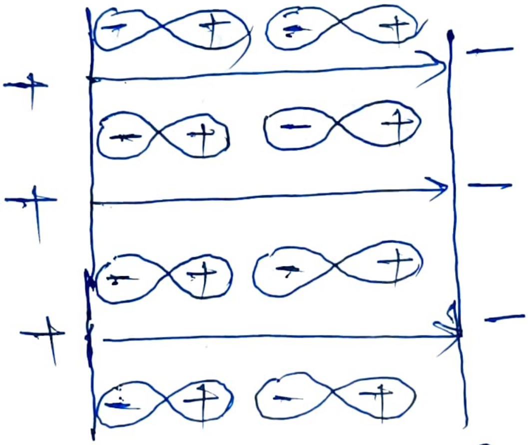
induced dipoles $E \neq 0$

when the center of gravity of positive and negative charges in the molecules of dielectric coincide, it is called non polar molecules, it has zero dipole moment eg. H_2, N_2 & O_2. If the dielectric with non polar molecules is placed in an electric field, the charge centers of a non polar molecules becomes displace from their equillibrium position. The molecules are then said to become polarised by the electric field and are called induced dipoles.

Polar molecules



E=0 permanent dipoles



dipole moment increased.

When the center of gravity of positive & negative charges are displaced from each other the molecules are calle polar molecule. It has permanent dipole moment.

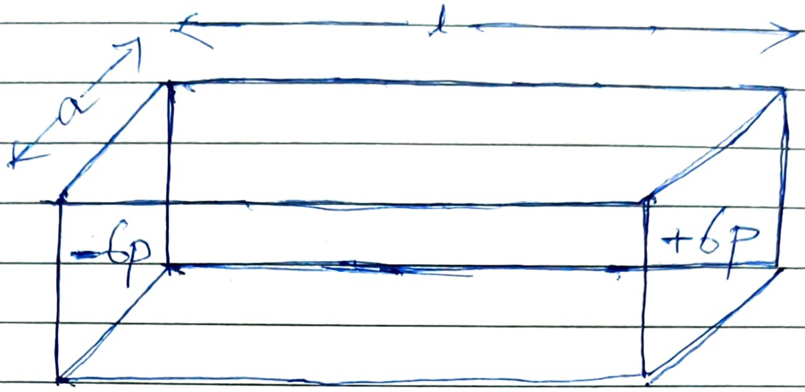
eg. H_2O, & N_2O

In the absense of external electric field dipoles are randomly oriented. when electric field is applied the forces on the dipole gives rise to couple whose effect is to orient the dipole along the direction of electric field, stronger the electric field greater will be alligning effect. polar molecules are oriented by the electric field and therefore their dipole moment increased.

The orientation of induced dipoles or permanant dipoles in an external electric field is such as to set the axis of dipole along the electric field is known as electric polarization.

Page No.:____
Date:__/__/__

Polarization of charges in dielectrics.



$\vec{E}$

When the dielectric slab placed in an electric field, electric field exert a force on each charge particle and dis-placing it from their equilibrium position in a opposite direction producing an electric dipole and dielectric is said to be polarised.

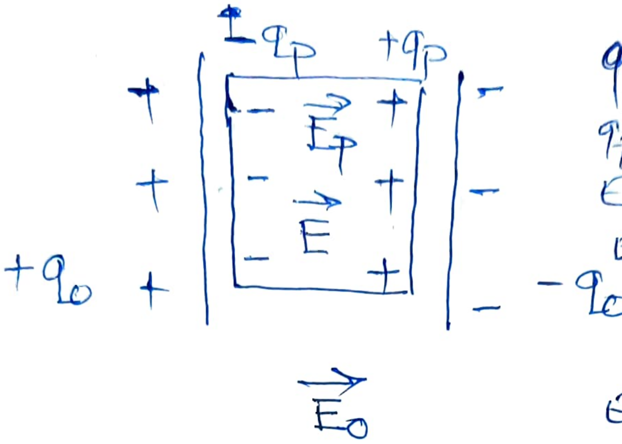
If the dipole moment in the dipole thus induced is p then the polarisation is defined as induced dipolemoment per unit vol. and is a vector quantity

$$\vec{P} = \frac{p}{V} = \frac{qd}{V} = \frac{Cm}{m^3} = \frac{C}{m^2}$$

Consider rectangular block of a dielectric material of length l and area of cross-section a. It is placed in electric field $\vec{E}$. The surface density of charge appea-ring on the faces of block are $+\sigma p$ and $-\sigma p$.

The electric polarization vector is defined as the surface density of charge appearing at a face perpendicular to the direction of applied field.

Relation between $\vec{D}$, $\vec{P}$ & $\vec{E}$



q_0 - real charges
q_p polarized charges
E_0 - ele field without dielectric
E_p - electric field due to polarizing charges
E - resultant electric field.

It is experimental found that in large number of dielectrics, polarisation is proportional to applied electric field $\vec{E}$

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} \qquad \text{——— (1)}$$

χ_e is scalar constant called electric susceptibility & ϵ_0 - permitivity of free space.

If a dielectric slab placed between the plate of condensor. The dielectric is polarised and bound charges q_p appearing on the dielectric surface & it is opposite to real charges q_0 present on the plates.

The net effect of presense of dielectric within the plates to reduce the electric field $\vec{E}_0$.

the resultant electric field is

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

where E_0 - is the electric field without dielectric & E_p is the electric field due to polarisation of charges.

the dielectric constant or relative permitivity is defined as

$$K = \epsilon_r = \frac{\text{Field in vacuum}}{\text{Field in dielectric}}$$

$$\epsilon_r = \frac{\vec{E_0}}{\vec{E}} = \frac{q/4\pi\epsilon_0 \cdot r^2}{q/4\pi\epsilon r^2}$$

$$\epsilon_r = \frac{1/\epsilon_0}{1/\epsilon} = \frac{\epsilon}{\epsilon_0} \quad \text{———} \quad (2)$$

$\therefore \vec{E} < \vec{E_0}$

$\epsilon > \epsilon_0$ so that $\epsilon_r > 1$

we can write

$$\epsilon_r = 1 + \chi_e \quad \text{———} \quad (3)$$

Since the electric field intensity depend upon medium. we define another electrical vector which depend upon magnitude of charges and its distribution but independant of the nature of medium. This vector is called electric displacement vector and is given by

$\vec{D} \propto \vec{E}$ or $\vec{D} = \epsilon \vec{E}$

$\because \epsilon = \epsilon_0 \epsilon_r$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad \text{———} \quad \cancel{B} \; (4)$$

using equation ~~A~~ (3) equation ~~B~~ (4) can ~~will~~ be written as

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E}$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E}$$

using equation (1)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

Is the relation between $\vec{D}$, $\vec{P}$ & $\vec{E}$ vectors.